A Theory of Legal Proof

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Thesis: Legal Proof is *Justified Belief* of Guilt

A defendant should be found guilty iff you (= the fact-finder) are justified to believe that the defendant is guilty based on the available and admissible evidence.

1. What is Justified Belief?
2. What is Statistical Evidence?
3. What are Legal Standards of Proof?

1. What is Justified Belief?

The Simple Threshold View

You are justified to believe a proposition A iff $P(A \mid E_t) > s$ for some fixed threshold s and your total evidence E_t.

> This view explains how your beliefs can be justified: your total evidence E_t justifies your high credence in A, which in turn justifies your belief in A.

> Example: Your credence is .96 that the defendant murdered Jane based on the available and admissible evidence.

> It seems you are justified to believe that the defendant murdered Jane.

> Legal probabilism: You should find the defendant guilty because he is *very probably guilty.* (Hedden and Colyvan, 2019).

> A—if not 'the'—problem for this view and legal probabilism alike is statistical evidence, which supports a high credence but no justified belief.

Prisoners Example I

Suppose there are 100 prisoners in a yard under the supervision of a guard. 99 of them join a pre-planned attack to kill the guard. One prisoner clearly refrains, standing alone in a corner. We know this from a reliable video recording. However, the video footage does not allow to discern the individual prisoners—all wear the same uniforms and the quality is not good enough to identify faces or other characteristics. There is no other evidence. Each prisoner is tried in a court of law.

Are you justified to believe that this prisoner standing trial—let's label him prisoner 1—is guilty?
- On the simple threshold view: yes because your credence in his guilt is high enough.
- If legal proof is high enough credence of guilt, prisoner 1 should be found guilty.
- But many are not willing to endorse this consequence (Wells, 1992; Redmayne, 2008; Blome-Tillmann, 2015).

Prisoners Example II

Why shouldn't you find the prisoner standing trial guilty?

> *we never think it justified to blame an individual on the basis of merely statistical evidence […] And this is best explained by the fact that we need a belief in someone's guilt to blame her, and that merely statistical evidence cannot give rise to a belief in these cases. (Buchak, 2014, p. 303)* ✓

- Buchak also says that full belief cannot be reduced to credence.
- And so she concludes: "threshold views of the relationship between *licensed court verdicts* and rational credence are false." (p. 291) ✗
- Similarly, Moss (2021, p. 11) writes: the "criminal standard of proof cannot be defined in terms of any threshold notion of confidence." ✗

Eye-witness Example

Imagine we had no video recording but the prison director walks by. She then testifies about the prisoner: "I saw him killing the guard!" Let's suppose you think that the director is very reliable but not perfectly so: she raises your credence that prisoner 1 is guilty to .99.

Should you find the prisoner guilty?

- Many think the eye-witness testimony suffices for a conviction.
- Given that the $P(Guilt) = .99$ in both examples, isn't it irrational to judge them differently?
- How should a high $P(Guilt)$ translate into a binary verdict?
- Statistical and individual evidence seem to differ in their support for a verdict of guilt—even if they make the defendant's guilt equally likely.

- Old: your credences are determined by your total evidence.
- New: your total evidence determines the possibilities you consider overall.

  You consider all and only those possibilities your credences assign a definite positive probability value.

  The total evidence you received partitions the underlying set W of all logical possibilities into possibilities π_i that are assigned a positive credence $P_\Pi(\{\pi_i\}) > 0$.[1]

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[1]A possibility π_i is a maximally specific way things might be with respect to the partition Π induced by your total evidence. A partition Π on W is a set of pairwise disjoint and non-empty subsets π_i of W so that $\bigcup \pi_i = W$.

You are justified to believe a proposition $A \subseteq \Pi$ iff you have a high credence $P_\Pi(A)$ and you expect it to remain more likely than not.[2]

- You expect a credence $P_\Pi(A)$ to remain more likely than not iff your conditional credence $P_\Pi(A \mid B) > 1/2$ for any proposition $B \subseteq \Pi$ you consider relevant. (cf. Leitgeb (2014))
- You consider a proposition $B \subseteq \Pi$ to be relevant to $A \subseteq \Pi$ iff $P_\Pi(B) > 0$ and $A \cap B \neq \emptyset$.

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[2]A proposition $A \subseteq \Pi$ is consistent iff $A \neq \emptyset$. A proposition A is consistent with a proposition B iff $A \cap B \neq \emptyset$. A entails B iff $A \subseteq B$. The negation $\neg A$ of a proposition is given by its complement $\Pi \setminus A$, the conjunction of $A \wedge B$ of two propositions by their intersection $A \cap B$, and the disjunction $A \vee B$ by their union $A \cup B$.

- π_i: the possibility where only prisoner i is innocent.

- $P(\{\pi_i\}) = 1/100$, $\Pi = \{\{\pi_1\}, ..., \{\pi_{100}\}\}$.

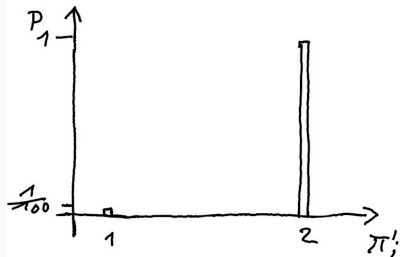- $\Pi \setminus \{\pi_1\}$: prisoner 1 is guilty; $P(\Pi \setminus \{\pi_1\}) = .99$.
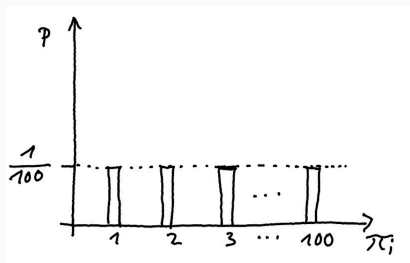
- Do you expect $\Pi \setminus \{\pi_1\}$ to remain more likely than not?

$$P(\Pi \setminus \{\pi_1\} \mid \{\pi_1, \pi_2\}) = \frac{P(\{\pi_2\})}{P(\{\pi_1, \pi_2\})} = 1/2.$$

- You are not justified to believe that prisoner 1 is guilty.

- $\Pi' = \{\{\pi_1'\}, \{\pi_2'\}\}$.
- $\Pi' \setminus \{\pi_1'\}$: prisoner 1 is guilty; $P(\Pi' \setminus \{\pi_1'\}) = .99$.

- Do you expect $\Pi' \setminus \{\pi_1'\}$ to remain more likely than not?

  $\{\pi_1'\}$ is inconsistent with $\Pi' \setminus \{\pi_1'\}$ and so irrelevant;
  $P(\{\pi_2'\} \mid \{\pi_2'\}) = 1$; and $P(\{\pi_2'\} \mid \Pi') = .99$.

- You are justified to believe that prisoner 1 is guilty.

Statistical Evidence

A Notion of Statistical Evidence

A piece of evidence is statistical iff it would assign a uniform probability distribution over the partition it induces if it were the only piece of evidence received.

Your total evidence is statistical iff it assigns uniform credences over the partition it induces.

- A piece of statistical evidence alone never gives rise to justified belief—as Buchak says.
- Our notion explains why "a base rate unaccompanied by other evidence" is statistical evidence (Koehler and Shaviro, 1990, p. 264).
- It also explains that non-uniform statistics are not statistical evidence. (Controversial!)
- 'The law' should have no aversion to probabilistic evidence, only to uniform distributions (Allen and Smiciklas, 2022).

Standards of Proof

Strongest Belief B_θ and Thresholds

Your threshold for justified belief *is* the strongest proposition B_θ you are justified to believe.

- Your state of justified belief is represented by B_θ: the conjunction of all the propositions you believe—the smallest set of possibilities of which you have a high enough credence and you expect it to remain more likely than not.
- You are justified to believe a proposition $A \subseteq \Pi$ iff $B_\theta \subseteq A$.
- The credence in B_θ is your threshold for justified belief: you are justified to believe $A \subseteq \Pi$ iff $P_\Pi(A) \geq P_\Pi(B_\theta)$.

Proof Beyond Reasonable Doubt $\overset{!}{=}$ Justified Belief in a Criminal Trial

A defendant's guilt should be proven beyond reasonable doubt iff your credence in his guilt is high enough and you expect it to remain more likely than not.

- You have a reasonable doubt if you consider a proposition to be relevant which would lower your credence in the defendant's guilt to $1/2$ or below.
- If you have a reasonable doubt, you consider a set of possibilities consistent with your beliefs which would make the defendant's innocence more likely than not.
- If so, your belief in guilt is not justified.

"It depends on the stakes." (Kaplan, 1968)

	Guilty	Innocent
finding guilty	*TG*	*FG*
finding innocent	*FI*	*TI*

- Decision theory: find guilty iff

$$P(G) \geq \frac{U(TI) - U(FG)}{U(TG) - U(FG) + U(TI) - U(FI)}.$$

- This gives us a constraint: pick B_θ such that $P(B_\theta) \geq P(G)$.
- A defendant should be found guilty beyond reasonable doubt iff $P(G) > P(B_\theta)$.
- (You expect B_θ to remain more likely than not and so must expect G to remain more likely than not.)

Preponderance of Evidence

"A plaintiff's claim counts as proven in court just in case the claim is established to be more likely than not."

	Guilty	Innocent
finding guilty	0	a
finding innocent	a	0

- In a civil trial, there is no preference for either finding for the plaintiff, or finding for the defendant: $U(FG) = U(FI)$.
- Hence, $P(G) = \dfrac{-u(FG)}{-u(FG) - u(FI)} = \dfrac{1}{2}$.
- A defendant should be found liable iff you expect G (here the defendant's liability) to remain more likely than not.
- This solves the 'paradox of the gatecrasher', a case of civil law that is structurally similar to the prisoners example (Blome-Tillmann, 2015).
- To believe by preponderance of evidence that the defendant is liable is thus identified with having a justified belief in a civil trial.

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Rational Reconstruction of a Trial Proceeding

1. Presumption of Innocence: $P(Guilt) = P(\neg Guilt) = 1/2$
   "normative prior" (Posner, 1999)
   - No finding of guilt without further evidence on our theory.
   - Fair to both parties.
   - Partial answer to the problem of the priors in the legal context.
2. Fix the utilities of true/false findings of guilt/innocence.
3. Updating on the total evidence E_t presented: $P_\Pi(\cdot \mid E_t) = x$
   - Induces partition Π.
4. Is it justified to believe the defendant is guilty in light of the credences P_Π based on E_t and the value judgments/utilities? If so, find guilty. If not, not.

Conclusion

A defendant should be found guilty just in case it is justified to believe that the defendant is guilty.

- *Pace* Buchak and Moss, our notion of justified belief implies a probabilistic threshold view that solves the problem posed by statistical evidence.
- Unlike other accounts, we need not impose any further condition on legal proof but justified belief (Enoch et al., 2012; Pritchard, 2015, 2018; Pardo, 2018; Smith, 2010, 2018; Blome-Tillmann, 2017; Moss, 2021).
- Proof beyond reasonable doubt should be justified belief in a criminal trial.
- Proof by preponderance of evidence should be rational belief in a civil trial.

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